

BIPOLARONIC PROXIMITY AND OTHER UNCONVENTIONAL EFFECTS IN CUPRATE SUPERCONDUCTORS

A. S. Alexandrov

*Department of Physics, Loughborough University,
Loughborough, United Kingdom*

a.s.alexandrov@lboro.ac.uk

Abstract There is compelling evidence for a strong electron-phonon interaction (EPI) in cuprate superconductors from the isotope effects on the supercarrier mass, high resolution angle resolved photoemission spectroscopies (ARPES), a number of optical and neutron-scattering measurements in accordance with our prediction of high-temperature superconductivity in polaronic liquids. A number of observations point to the possibility that high- T_c cuprate superconductors may not be conventional Bardeen-Cooper-Schrieffer (BCS) superconductors, but rather derive from the Bose-Einstein condensation (BEC) of real-space pairs, which are mobile small bipolarons. Here I review the bipolaron theory of unconventional proximity effects, the symmetry and checkerboard modulations of the order parameter and quantum magneto-oscillations discovered recently in cuprates.

Keywords: bipolarons, cuprates, proximity, symmetry, magnetooscillations

1. Polarons in high-temperature superconductors

Many unconventional properties of cuprate superconductors may be attributed to the Bose-Einstein condensation (BEC) of real-space pairs, which are mobile small bipolarons [1, 2, 3, 4, 5, 6, 7, 8]. A possible fundamental origin of such strong departure of the cuprates from conventional BCS behaviour is the unscreened Fröhlich EPI providing the polaron level shift E_p of the order of 1 eV [4], which is routinely neglected in the Hubbard U and $t - J$ models. This huge interaction with c -axis polarized optical phonons is virtually unscreened at any doping of cuprates. Even acting alone it leads to large bipolarons in the continuum limit, if the ratio $\eta = \epsilon/\epsilon_0$ of the high-frequency (electronic) and static dielectric

constants is small enough [3]. The large Fröhlich bipolarons are further stabilised in going from 3D to 2D [9]. When acting together with the deformation potential and the Jahn-Teller EPIs the Fröhlich EPI overcomes the inter-site Coulomb repulsion forming small bipolarons [5]. Hence, in order to build the adequate theory of high-temperature superconductivity, the Coulomb repulsion and the *unscreened* long-range EPI together with the short-range one should be treated on an equal footing with the short-range repulsive Hubbard U . When these interactions are strong compared with the kinetic energy of carriers, this "Coulomb-Fröhlich" model predicts the ground state in the form of superlight small bipolarons [4, 10, 11].

Nowadays compelling evidence for a strong EPI has arrived from isotope effects [12, 13], more recent high resolution angle resolved photoemission spectroscopies (ARPES) [14], and a number of earlier optical [15, 16, 17, 18, 19], neutron-scattering [20] and recent inelastic scattering studies [21] of cuprates and related compounds. Whereas calculations based on the local spin-density approximation (LSDA) often predict negligible EPI, the inclusion of Hubbard U in the $LSDA + U$ calculations greatly enhances its strength [22].

A parameter-free estimate of the Fermi energy using the magnetic-field penetration depth found a very low value, $\epsilon_F \lesssim 100$ meV [23] clearly supporting the real-space (i.e individual) pairing in cuprate superconductors. There is strong experimental evidence for a gap in the normal-state electron density of states of cuprates [2], which is known as the pseudogap. Experimentally measured pseudogaps of many cuprates are $\gtrsim 50$ meV [24]. If following Ref. [25] one accepts that the pseudogap is about half of the pair binding energy, Δ , then the condition for real-space pairing, $\epsilon_F \lesssim \pi\Delta$, is well satisfied in most cuprates (typically the small bipolaron radius is $r_b \approx 0.2 - 0.4$ nm).

Also magnetotransport and thermal magnetotransport data strongly support preformed bosons in cuprates. In particular, many high-magnetic-field studies revealed a non-BCS upward curvature of the upper critical field $H_{c2}(T)$ (see [26] for a review of experimental data), in accordance with the theoretical prediction for the Bose-Einstein condensation of charged bosons in the magnetic field [27]. The Lorenz number, $L = e^2\kappa_e/T\sigma$ differs significantly from the Sommerfeld value $L_e = \pi^2/3$ of the standard Fermi-liquid theory, if carriers are double-charged bosons [28]. Here κ_e , and σ are electron thermal and electrical conductivities, respectively. Ref. [28] predicted a rather low Lorenz number for bipolarons, $L \approx 0.15L_e$, due to the double elementary charge of bipolarons, and also due to their nearly classical distribution function above T_c . Direct measurements of the Lorenz number using the thermal Hall effect

[29] produced the value of L just above T_c about the same as predicted by the bipolaron model, and its strong temperature dependence. This breakdown of the Wiedemann-Franz law is apparently caused by excited single polarons coexisting with bipolarons in the thermal equilibrium [30, 31]. Also unusual normal state diamagnetism uncovered by torque magnetometry has been convincingly explained as the normal state (Landau) diamagnetism of charged bosons [32].

However, despite clear evidence for the existence of polarons in cuprates, no consensus currently exists concerning the microscopic mechanism of high-temperature superconductivity. While a number of early (1990s) and more recent studies prove that the Mott-Hubbard insulator promotes doping-induced polaron formation [33], some other works suggest that EPI does not only not help, but hinder the pairing instability. The controversy should be resolved experimentally. Here I argue that the giant (GPE) and nil (NPE) proximity effects provide another piece of evidence for bipolaronic BEC in cuprates [34]. The same bipolaronic scenario also explains the symmetry and checkerboard modulations of the order parameter, and recently observed magnetooscillations in the vortex state of cuprates.

2. Unconventional proximity effects

Several groups [35] reported that in the Josephson cuprate SNS junctions supercurrent can run through normal N -barriers with the thickness $2L$ greatly exceeding the coherence length, when the barrier is made from a slightly doped non-superconducting cuprate (the so-called N' barrier), Fig.1.

Using the advanced molecular beam epitaxy, Bozovic *et al.* [35] proved that GPE is intrinsic, rather than caused by any extrinsic inhomogeneity of the barrier. Resonant scattering of soft-x-ray radiation did not find any signs of intrinsic inhomogeneity (such as charge stripes, charge-density waves, etc.) either [36]. Hence GPE defies the conventional explanation, which predicts that the critical current should exponentially decay with the characteristic length of about the coherence length, $\xi \lesssim 1$ nm in cuprates. Annealing the junctions at low temperatures in vacuum rendered the barrier insulating. Remarkably when the $SN'S$ junction was converted into a superconductor-insulator-superconductor (SIS) device no supercurrent was observed, even in devices with the thinnest (one unit cell thick) barriers [37] (nil proximity effect, NPE).

Both GPE and NPE can be broadly understood as the bipolaronic Bose-condensate tunnelling into a cuprate *semiconductor* [34].

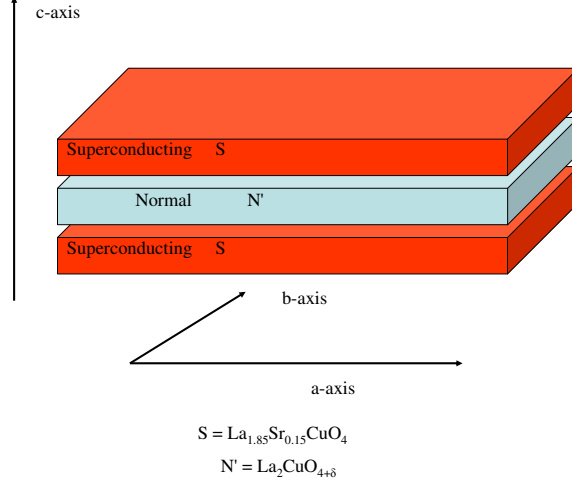


Figure 1. SNS cuprate nanostructure

To illustrate the point one can apply the Gross-Pitaevskii (GP)-type equation for the superconducting order parameter $\psi(\mathbf{r})$, generalized by us [38] for a charged Bose liquid (CBL), since, as discussed above, many observations including a small coherence length point to a possibility that cuprate superconductors may not be conventional BCS superconductors, but rather derive from BEC of real-space pairs, such as mobile small bipolarons [4],

$$\left[E(-i\hbar\nabla + 2e\mathbf{A}) - \mu + \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}') |\psi(\mathbf{r}')|^2 \right] \psi(\mathbf{r}) = 0. \quad (1)$$

Here $E(\mathbf{K})$ is the center-of-mass pair dispersion and the Peierls substitution, $\mathbf{K} \Rightarrow -i\hbar\nabla + 2e\mathbf{A}$ is applied with the vector potential $\mathbf{A}(\mathbf{r})$.

The integro-differential equation (1) is quite different from the Ginzburg-Landau [39] and Gross-Pitaevskii [40] equations, describing the order parameter in the BCS and neutral superfluids, respectively. Here μ is the chemical potential and $V(\mathbf{r})$ accounts for the long-range Coulomb and a short-range composed boson-boson repulsions [5]. While the electric field potential can be found from the corresponding Poisson-like equation [38], a solution of two coupled nonlinear differential equations for the order parameter $\psi(\mathbf{r})$ and for the potential $V(\mathbf{r})$ in the nanostructure, Fig.1, is a nontrivial mathematical problem. For more transparency we restrict our analysis in this section by a short-range poten-

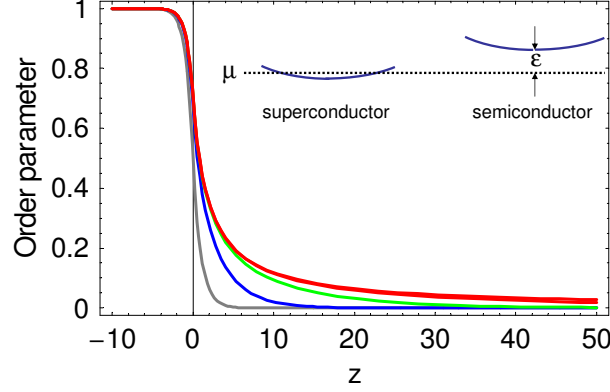


Figure 2. BEC order parameter at the SN boundary for $\tilde{\mu} = 1.0, 0.1, 0.01$ and ≤ 0.001 (upper curve). The chemical potential is found above the boson band-edge due to the boson-boson repulsion in cuprate superconductors and below the edge in cuprate semiconductors with low doping.

tial, $V(\mathbf{r}) = v|\psi(\mathbf{r})|^2$, where a constant v accounts for the short-range repulsion. Then in the absence of the magnetic field Eq.(1) is reduced to the familiar GP equation [40] using the continuum (effective mass) approximation, $E(\mathbf{K}) = K^2/2m_c$. In the tunnelling geometry of $SN'S$ junctions, Figs.1,2, it takes the form,

$$\frac{1}{2m_c} \frac{d^2\psi(Z)}{dZ^2} = [v|\psi(Z)|^2 - \mu]\psi(Z), \quad (2)$$

in the superconducting region, $Z < 0$, Fig.2. Here m_c is the boson mass in the direction of tunnelling along Z ($\hbar = c = k_B = 1$ in this section). Deep inside the superconductor the order parameter is a constant, $|\psi(Z)|^2 = n_s$ and $\mu = vn_s$, where the condensate density n_s is about $n_s \approx x/2$, if the temperature is well below T_c of the superconducting electrode. Here the in-plane lattice constant a and the unit cell volume are taken as unity, and x is the doping level as in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

The normal barrier at $Z > 0$ is an underdoped cuprate above its transition temperature, $T'_c < T$ where the chemical potential μ lies below the bosonic band by some energy ϵ , Fig.2, found from $\int dE N(E) [\exp((E + \epsilon)/T) - 1]^{-1} = x'/2$. Here $N(E)$ is the bipolaron density of states (DOS),

and $x' < x$ is the doping level of the barrier. In-plane bipolarons are quasi-two dimensional repulsive bosons, propagating along the CuO_2 planes with the effective mass m several orders of magnitude smaller than their out-of-plane mass, $m_c \gg m$, [5]. Using bipolaron band dispersion, $E(\mathbf{K}) = K^2/2m + 2t_c[1 - \cos(K_\perp d)]$, the density of states is found as $N(E) = (m/2\pi^2) \arccos(1 - E/2t_c)$ for $0 < E < 4t_c$, and $N(E) = m/2\pi$ for $4t_c < E$. Here K and K_\perp are the in-plane and out-of-plane center-of-mass momenta, respectively, $t_c = 1/2m_c d^2$, and d is the inter-plane distance. As a result one obtains

$$\epsilon(T) \leq -T \ln(1 - e^{-T_0/T}), \quad (3)$$

which is exponentially small at $T'_c < T \ll T_0$ turning into zero at $T = T'_c$, where $T'_c \approx T_0/\ln(T_0/2t_c)$, and $T_0 = \pi x'/m \gg T'_c \gg t_c$.

It is important to note that $\epsilon(T)$ remains also small at $T'_c(2D) \leq T \ll T_0$ in the purely two-dimensional repulsive Bose-gas [41]. While in two dimensions Bose condensation does not occur in either the ideal or the interacting system, there is a phase transition to a superfluid state at $T'_c(2D) = T_0/\ln(1/f_0) \ll T_0$, where $f_0 \ll 1$ depends on the density of hard-core dilute bosons and their repulsion [42, 41]. The superfluid transition takes place only if there is a residual repulsion between bosons, i.e. $T'_c(2D) = 0$ for the ideal 2D Bose-gas. Actually $T'_c(2D)$ gives a very good estimate for the exact Berezinski-Kosterlitz-Thouless (BKT) critical temperature in the dilute Bose gas, where the BKT contribution of vortices is important only very close to $T'_c(2D)$ [42].

The GP equation in the barrier can be written as

$$\frac{1}{2m_c} \frac{d^2\psi(Z)}{dZ^2} = [v|\psi(Z)|^2 + \epsilon]\psi(Z). \quad (4)$$

Introducing the bulk coherence length, $\xi = 1/(2m_c n_s v)^{1/2}$ and dimensionless $f(z) = \psi(Z)/n_s^{1/2}$, $\tilde{\mu} = \epsilon/n_s v$, and $z = Z/\xi$ one obtains for a real $f(z)$

$$\frac{d^2 f}{dz^2} = f^3 - f, \quad (5)$$

if $z < 0$, and

$$\frac{d^2 f}{dz^2} = f^3 + \tilde{\mu} f, \quad (6)$$

if $z > 0$. These equations can be readily solved using first integrals of motion respecting the boundary conditions, $f(-\infty) = 1$, and $f(\infty) = 0$,

$$\frac{df}{dz} = -(1/2 + f^4/2 - f^2)^{1/2}, \quad (7)$$

and

$$\frac{df}{dz} = -(\tilde{\mu}f^2 + f^4/2)^{1/2}, \quad (8)$$

for $z < 0$ and $z > 0$, respectively. The solution in the superconducting electrode is given by

$$f(z) = \tanh \left[-2^{-1/2}z + 0.5 \ln \frac{2^{1/2}(1 + \tilde{\mu})^{1/2} + 1}{2^{1/2}(1 + \tilde{\mu})^{1/2} - 1} \right]. \quad (9)$$

It decays in the close vicinity of the barrier from 1 to $f(0) = [2(1 + \tilde{\mu})]^{-1/2}$ in the interval about the coherence length ξ . On the other side of the boundary, $z > 0$, it is given by

$$f(z) = \frac{(2\tilde{\mu})^{1/2}}{\sinh\{z\tilde{\mu}^{1/2} + \ln[2(\tilde{\mu}(1 + \tilde{\mu}))^{1/2} + (1 + 4\tilde{\mu}(1 + \tilde{\mu}))^{1/2}]\}}. \quad (10)$$

Its profile is shown in Fig.2. Remarkably, the order parameter penetrates the normal layer up to the length $Z^* \approx (\tilde{\mu})^{-1/2}\xi$, which could be larger than ξ by many orders of magnitude, if $\tilde{\mu}$ is small. It is indeed the case, if the barrier layer is sufficiently doped. For example, taking $x' = 0.1$, c-axis $m_c = 2000m_e$, in-plane $m = 10m_e$, $a = 0.4$ nm, and $\xi = 0.6$ nm, yields $T_0 \approx 140$ K and $(\tilde{\mu})^{-1/2} \gtrsim 50$ at $T = 25$ K. Hence the order parameter could penetrate the normal cuprate semiconductor up to a hundred coherence lengths or even more as observed (GPE) [35]. If the thickness of the barrier L is small compared with Z^* , and $(\tilde{\mu})^{1/2} \ll 1$, the order parameter decays following the power law, rather than exponentially,

$$f(z) = \frac{\sqrt{2}}{z + 2}. \quad (11)$$

Hence, for $L \lesssim Z^*$, the critical current should also decay following the power law rather than exponentially. On the other hand, for the *undoped* barrier $\tilde{\mu}$ becomes larger than unity, $\tilde{\mu} \propto \ln(mT/\pi x') \rightarrow \infty$ for any finite temperature T when $x' \rightarrow 0$, and the current should exponentially decay with the characteristic length smaller than ξ , which is experimentally observed as well (NPE) [37].

3. Quantum magneto-oscillations, d-wave symmetry and checkerboard modulations

Until recently no convincing signatures of quantum magneto-oscillations have been found in the normal state of cuprate superconductors despite significant experimental efforts. There are no normal state oscillations even in high quality single crystals of overdoped cuprates like

$\text{Ti}_2\text{Ba}_2\text{CuO}_6$, where conditions for de Haas-van Alphen (dHvA) and Shubnikov-de Haas (SdH) oscillations seem to be perfectly satisfied [43] and a large Fermi surface is identified in the angle-resolved photoemission spectra (ARPES) [44]. The recent observations of magneto-oscillations in kinetic [45, 46] and magnetic [47, 48] response functions of underdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6.5}$ and $\text{YBa}_2\text{Cu}_4\text{O}_8$ are perhaps even more striking since many probes of underdoped cuprates including ARPES [49] clearly point to a non Fermi-liquid normal state. Their description in the framework of the standard theory for a metal [50] has led to a very small Fermi-surface area of a few percent of the first Brillouin zone [45, 46, 47, 48], and to a low Fermi energy of only about *the room temperature* [47]. Clearly such oscillations are incompatible with the first-principle (LDA) band structures of cuprates, but might be compatible with a low Fermi energy and non-adiabatic polaronic normal state of charge-transfer Mott insulators as discussed in section 1.

Nevertheless one can raise a doubt concerning their normal state origin. The magnetic length, $\lambda \equiv (\pi\hbar/eB)^{1/2} \gtrsim 5$ nm, remains larger than the zero-temperature in-plane coherence length, $\xi \lesssim 2$ nm, measured independently, in any field reached in Ref. [45, 46, 47, 48]. Hence the magneto-oscillations are observed in the vortex (mixed) state well below the upper critical field, rather than in the normal state, as also confirmed by the *negative* sign of the Hall resistance [45]. It is well known, that in "YBCO" the Hall conductivities of vortexes and quasiparticles have opposite sign causing the sign change in the Hall effect in the mixed state [51]. Also there is a substantial magnetoresistance [46], which is a signature of the flux flow regime rather than of the normal state. Hence it would be rather implausible if such oscillations have a normal-state origin due to small electron Fermi surface pockets [48] with the characteristic wave-length of electrons larger than the widely accepted coherence length.

Here I propose an alternative explanation of the magneto-oscillations [45, 46, 47, 48] as emerging from the quantum interference of the vortex lattice and the checkerboard or lattice modulations of the order parameter observed by STM with atomic resolution [52]. The checkerboard effectively pins the vortex lattice, when the period of the latter, λ is commensurate with the period of the checkerboard lattice, a . The condition $\lambda = Na$, where N is a large integer, yields $1/B^{1/2}$ periodicity of the response functions, rather than $1/B$ periodicity of conventional normal state magneto-oscillations.

The integro-differential equation (1) in the continuum approximation, $E(\mathbf{K}) = \hbar^2 K^2/2m$, with the long-range Coulomb repulsion between double charged bosons, $V(\mathbf{r}) = V_c(\mathbf{r}) = 4e^2/\epsilon_0 r$, describes a single vortex

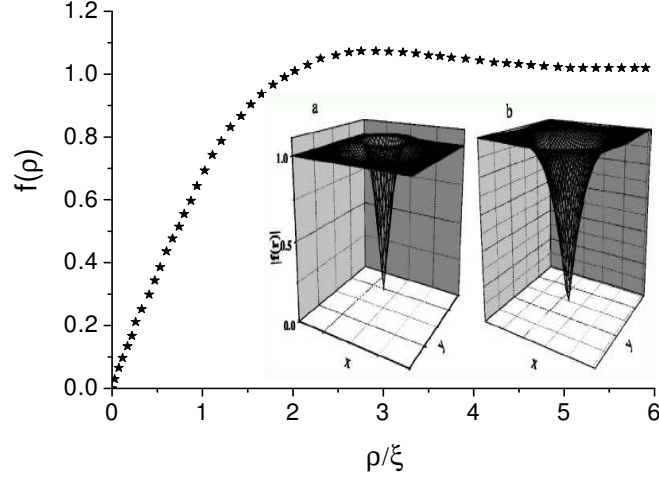


Figure 3. The order parameter profile $f(\rho) = \psi(\mathbf{r})/n_s^{1/2}$ of a single vortex in CBL [38] (symbols). Inset: CBL vortex (a) [38, 54] compared with the Abrikosov vortex (b) [53] (here $\rho = [x^2 + y^2]^{1/2}$).

with a *charged* core, Fig.3, when the magnetic field, B is applied. The coherence length in this case is roughly the same as the screening radius, $\xi = (\hbar/2^{1/2}m\omega_p)^{1/2}$. Here $\omega_p = (16\pi n_s e^2/\epsilon_0 m)^{1/2}$ is the CBL plasma frequency, ϵ_0 the static dielectric constant of the host lattice, m is the (in-plane) boson mass, and n_s is the average condensate density. The chemical potential is zero, $\mu = 0$, if one takes into account the Coulomb interaction alone due to a neutralizing homogeneous charge background. Each vortex carries one flux quantum, $\phi_0 = \pi\hbar/e$, but it has an unusual core, Fig.3a, Ref. [38], due to a local charge redistribution caused by the magnetic field, different from the conventional vortex [53], Fig.3b. Remarkably, the coherence length turns out very small, $\xi \approx 0.5\text{nm}$ with the material parameters typical for underdoped cuprates, $m = 10m_e$, $n_s = 10^{21}\text{cm}^{-3}$ and $\epsilon_0 = 100$.

The coherence length ξ is so small at low temperatures, that the distance between two vortices remains large compared with the vortex size, $\lambda \gg \xi$, in any laboratory field reached so far [45, 46, 47, 48]. It allows us to write down the vortex-lattice order parameter, $\psi(\mathbf{r}) =$

$\psi_{vl}(\mathbf{r})$, as

$$\psi_{vl}(\mathbf{r}) \approx n_s^{1/2} \left[1 - \sum_j \phi(\mathbf{r} - \mathbf{r}_j) \right], \quad (12)$$

where $\phi(\mathbf{r}) = 1 - f(\rho)$, and $\mathbf{r}_j = \lambda\{n_x, n_y\}$ with $n_{x,y} = 0, \pm 1, \pm 2, \dots$ (if, for simplicity, we take the square vortex lattice [55]). The function $\phi(\rho)$ is linear well inside the core, $\phi(\rho) \approx 1 - 1.52\rho/\xi$ ($\rho \ll \xi$), and it has a small negative tail, $\phi(\rho) \approx -4\xi^4/\rho^4$ outside the core when $\rho \gg \xi$, Fig.3 [38].

In the continuum approximation with the Coulomb interaction alone the magnetization of CBL follows the standard logarithmic law, $M(B) \propto \ln 1/B$ without any oscillations since the magnetic field profile is the same as in the conventional vortex lattice [54]. However, more often than not the center-of-mass Bloch band of preformed pairs, $E(\mathbf{K})$, has its minima at some finite wave vectors $\mathbf{K} = \mathbf{G}$ of their center-of-mass Brillouin zone [4, 5]. Near the minima the GP equation (1) is written as

$$\left[\frac{(-i\hbar\nabla - \hbar\mathbf{G} + 2e\mathbf{A})^2}{2m^{**}} - \mu \right] \psi(\mathbf{r}) + \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}') |\psi(\mathbf{r}')|^2 \psi(\mathbf{r}) = 0, \quad (13)$$

with the solution $\psi(\mathbf{r}) = \psi_{\mathbf{G}}(\mathbf{r}) \equiv e^{i\mathbf{G}\cdot\mathbf{r}} \psi_{vl}(\mathbf{r})$, if the interaction is the long-range Coulomb one, $V(\mathbf{r}) = V_c(\mathbf{r})$.

In particular, a nearest-neighbor (nn) approximation for the hopping of intersite bipolarons between oxygen p-orbitals on the CuO_2 2D lattice yields four generate states $\psi_{\mathbf{G}}$ with $\mathbf{G}_i = \{\pm 2\pi/a_0, \pm 2\pi/a_0\}$, where a_0 is the lattice period [10]. Their positions in the Brillouin zone move towards Γ point beyond the nn approximation. The true ground state is a superposition of four degenerate states, respecting time-reversal and parity symmetries [56],

$$\psi(\mathbf{r}) = A n_s^{1/2} [\cos(\pi x/a) \pm \cos(\pi y/a)] \psi_{vl}(\mathbf{r}). \quad (14)$$

Here we use the reference frame with x and y axes along the nodal directions and $a = 2^{-3/2}a_0$. Two "plus/minus" coherent states, Eq.(14), are physically identical since they are related via a translation transformation, $y \Rightarrow y + a$. Normalizing the order parameter by its average value $\langle \psi(\mathbf{r})^2 \rangle = n_s$ and using $(\xi/\lambda)^2 \ll 1$ as a small parameter yield the following "minus" state amplitude, $A \approx 1 - N \sum_{n=0}^{\infty} 2[\tilde{\phi}_1(2^{1/2}\pi/a) + \tilde{\phi}_2(2^{1/2}\pi/a)]\delta_{n,R/2} + [\tilde{\phi}_1(2\pi/a) + \tilde{\phi}_2(2\pi/a)]\delta_{n,R}$ for the square vortex lattice [55] with the reciprocal vectors $\mathbf{g} = (2\pi/\lambda)\{n_x, n_y\}$. Here $\delta_{n,R}$ is the Kroneker symbol, $R = \lambda/a$ is the ratio of the vortex lattice

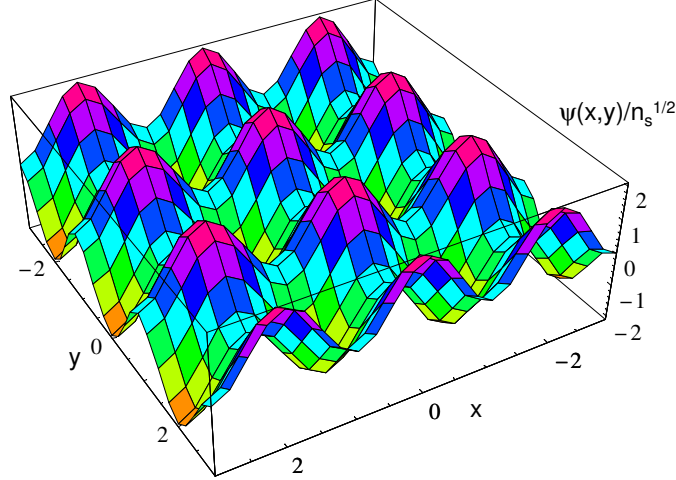


Figure 4. The checkerboard d-wave order parameter of CBL [56] on the square lattice in zero magnetic field (coordinates x, y are measured in units of a).

period to the checkerboard period ($n = 0, 1, 2, \dots$), $N = BS/\phi_0$ is the number of flux quanta in the area S of the sample, and $\tilde{\phi}_k(q) = (2\pi/S) \int_0^\infty d\rho \rho J_0(\rho q) \phi^k(\rho)$ is the Fourier transform of k 's power of $\phi(\rho)$, where $J_0(x)$ is the zero-order Bessel function.

The order parameter $\psi(\mathbf{r})$, Eq.(14) has the d -wave symmetry changing sign in real space, when the lattice is rotated by $\pi/2$. This symmetry is due to the pair center-of-mass energy dispersion with the four minima at $\mathbf{K} \neq 0$, rather than due to a specific symmetry of the pairing potential. It also reveals itself as a *checkerboard* modulation of the carrier density with two-dimensional patterns in zero magnetic field, Fig.4, as predicted by us [56] prior to their observations [52]. Solving the Bogoliubov-de Gennes equations with the order parameter, Eq.(14), yields the real-space checkerboard modulations of the single-particle density of states [56], similar to those observed by STM in cuprate superconductors.

Now we take into account that the interaction between composed pairs includes a short-range repulsion along with the long-range Coulomb one, $V(\mathbf{r}) = V_c(\mathbf{r}) + v\delta(\mathbf{r})$ [5]. At sufficiently low carrier density the short-range repulsion can be treated as a perturbation to the ground state, Eq.(14). Importantly the short-range repulsion energy of CBL,

$U = (v/2)\langle\psi(\mathbf{r})^4\rangle$, has a part, ΔU , oscillating with the magnetic field as

$$\frac{\Delta U}{U_0} \approx N \sum_{n=0}^{\infty} [A_1 \delta_{n,R/2} + A_2 \delta_{n,R} + A_3 \delta_{n,2R}], \quad (15)$$

where $U_0 = vn_s^2/2$ is the hard-core energy of a homogeneous CBL, and the amplitudes are proportional to the Fourier transforms of $\phi(\rho)$ as

$$\begin{aligned} A_1 = & 15\tilde{\phi}_1 \left(2^{1/2}\pi/a \right) - 45\tilde{\phi}_2 \left(2^{1/2}\pi/a \right) + 24\tilde{\phi}_3 \left(2^{1/2}\pi/a \right) - 6\tilde{\phi}_4 \left(2^{1/2}\pi/a \right) \\ & + 8\tilde{\phi}_1 \left(10^{1/2}\pi/a \right) - 12\tilde{\phi}_2 \left(10^{1/2}\pi/a \right) + 8\tilde{\phi}_3 \left(10^{1/2}\pi/a \right) - 2\tilde{\phi}_4 \left(10^{1/2}\pi/a \right), \end{aligned} \quad (16)$$

$$\begin{aligned} A_2 = & -(23/2)\tilde{\phi}_1 (2\pi/a) + (57/2)\tilde{\phi}_2 (2\pi/a) - 16\tilde{\phi}_3 (2\pi/a) + 4\tilde{\phi}_4 (2\pi/a) \\ & - 12\tilde{\phi}_1 (2^{3/2}\pi/a) + 9\tilde{\phi}_2 (2^{3/2}\pi/a) - 6\tilde{\phi}_3 (2^{3/2}\pi/a) + 3\tilde{\phi}_4 (2^{3/2}\pi/a) \end{aligned} \quad (17)$$

$$A_3 = -\tilde{\phi}_1 (4\pi/a) + (3/2)\tilde{\phi}_2 (4\pi/a) - \tilde{\phi}_3 (4\pi/a) + (1/4)\tilde{\phi}_4 (4\pi/a). \quad (18)$$

Fluctuations of the pulsed magnetic field and unavoidable disorder in cuprates induce some random distribution of the vortex-lattice period, λ . Hence one has to average ΔU over R with the Gaussian distribution, $G(R) = \exp[-(R - \bar{R})^2/\gamma^2]/\gamma\pi^{1/2}$ around an average \bar{R} with the width $\gamma \ll \bar{R}$. Then using the Poisson summation formula yields

$$\begin{aligned} \frac{\Delta U}{U_0} = & N \sum_{k=0}^{\infty} A_1 e^{-\pi^2 k^2 \gamma^2 / 16} \cos(\pi k \bar{R}) \\ & + A_2 e^{-\pi^2 k^2 \gamma^2 / 4} \cos(2\pi k \bar{R}) + A_3 e^{-\pi^2 k^2 \gamma^2} \cos(4\pi k \bar{R}). \end{aligned} \quad (19)$$

The oscillating correction to the magnetic susceptibility, $\Delta\chi(B) = -\partial^2 \tilde{\Omega} / \partial B^2$, is strongly enhanced due to high oscillating frequencies in Eq.(19). Since the superfluid has no entropy we can use ΔU as the quantum correction to the thermodynamic potential $\tilde{\Omega}$ even at finite temperatures below $T_c(B)$. Differentiating twice the first harmonic ($k = 1$) of the first lesser damped term in Eq.(19) we obtain

$$\Delta\chi(B) \approx \chi_0 e^{-\delta^2 B_0 / 16B} \left(\frac{B_0}{B} \right)^2 \cos(B_0/B)^{1/2}, \quad (20)$$

where $\chi_0 = U_0 S A_1 e^2 a^2 / 4\pi^4 \hbar^2$ is a temperature-dependent amplitude, proportional to the condensate density squared, $B_0 = \pi^3 \hbar / e a^2 = 8\pi^3 \hbar / e a_0^2$ is a characteristic magnetic field, which is approximately $1.1 \cdot 10^6$ Tesla

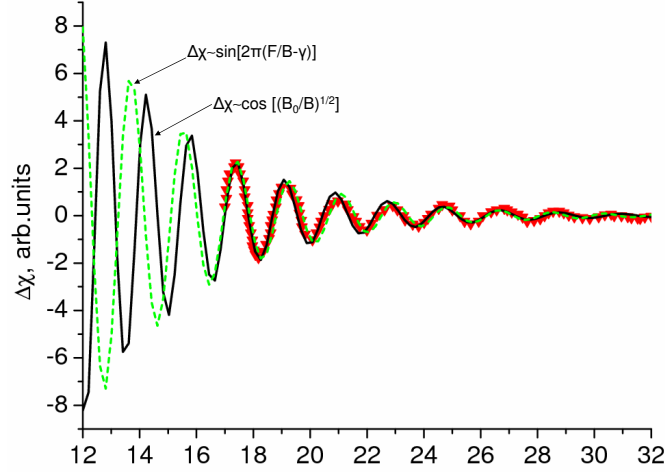


Figure 5. Quantum corrections to the vortex-lattice susceptibility versus $1/B$, Eq.(20) (solid line, $B_0 = 1.000 \cdot 10^6$ Tesla, $\delta = 0.06$) compared with oscillating susceptibility of $\text{YBa}_2\text{Cu}_3\text{O}_{6.5}$ (symbols) and with the conventional normal state oscillations (dashed line) [48] at $T = 0.4$ K.

for $a_0 \approx 0.38$ nm, and γ is replaced by $\gamma \equiv \delta \bar{R}$ with the relative distribution width δ . Assuming that $\xi \gtrsim a$, so that the amplitude A_1 is roughly a^2/S , the quantum correction $\Delta\chi$, Eq.(20), is of the order of $w x^2/B^2$, where x is the density of holes per unit cell. It is smaller than the conventional normal state (de Haas-van Alphen) correction, $\Delta\chi_{dHvA} \sim \mu/B^2$ [50], for a comparable Fermi-energy scale $\mu = wx$, since $x \ll 1$ in the underdoped cuprates.

Different from normal state dHvA oscillations, which are periodic versus $1/B$, the vortex-lattice oscillations, Eq.(20) are periodic versus $1/B^{1/2}$. They are quasi-periodic versus $1/B$ with a field-dependent frequency $F = B_0(B/B_0)^{1/2}/2\pi$, which is strongly reduced relative to the conventional-metal frequency ($\approx B_0/2\pi$) since $B \ll B_0$, as observed in the experiments [45, 46, 47, 48]. The quantum correction to the susceptibility, Eq.(20) fits well the oscillations in $\text{YBa}_2\text{Cu}_3\text{O}_{6.5}$ [48], Fig.5. The oscillations amplitudes, proportional to $n_s^2 \exp(-\delta^2 B_0/16B)$ decay with increasing temperature since the randomness of the vortex lattice, δ , increases, and the Bose-condensate evaporates.

4. Summary

A possibility of real-space pairing, proposed originally by Ogg [57] and later on by Schafroth and Blatt and Butler [58] has been the subject of many discussions as opposed to the Cooper pairing, particularly heated over the last 20 years after the discovery of high temperature superconductivity in cuprates. Our extension of the BCS theory towards the strong interaction between electrons and ion vibrations [5] proved that BCS and Ogg-Schafroth pictures are two extreme limits of the same problem. Cuprates are characterized by poor screening of high-frequency optical phonons, which allows the long-range Fröhlich EPI bound holes in superlight small bipolarons [4], which are several orders of magnitude lighter than small Holstein (bi)polareons.

The bipolaron theory accounts for GPE and NPE in slightly doped semiconducting and undoped insulating cuprates, respectively. It predicts the occurrence of a new length scale, $\hbar/\sqrt{2m_c\epsilon(T)}$, which turns out much larger than the zero-temperature coherence length in a wide temperature range above the transition point of the normal barrier, if bosons are almost two-dimensional (2D). The physical reason, why the quasi-2D bosons display a large normal-state coherence length, whereas 3D Bose-systems (or any-D Fermi-systems) at the same values of parameters do not, originates in the large DOS near the band edge of two-dimensional bosons compared with 3D DOS. Since DOS is large, the chemical potential is pinned near the edge with the magnitude, $\epsilon(T)$, which is exponentially small

Here I also propose that the magneto-oscillations in underdoped cuprate superconductors result from the quantum interference of the vortex lattice and the lattice modulations of the d-wave order parameter, Fig.4, which play the role of a periodic pinning grid. Our expression (20) describes the oscillations as well as the standard Lifshitz-Kosevich formula of dHvA and SdH effects [45, 46, 47, 48]. The difference of these two dependencies could be resolved in ultrahigh magnetic fields as shown in Fig.5. While our theory utilizes GP-type equation for composed charged bosons [38], the quantum interference of vortex and crystal lattice modulations of the order parameter is quite universal extending well beyond Eq.(1) independent of a particular pairing mechanism. It can also take place in the standard BCS superconductivity at $B < H_{c2}$, but hardly be observed because of much lower value of H_{c2} in conventional superconductors resulting in a very small damping factor, $\propto \exp(-\delta^2 B_0/16B) \lll 1$.

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